

Turnpoint Discrepancies in Competition Scoring



Summary

The various scoring programs used in hang gliding/paragliding competition calculate distances between two points on the Earth's surface using two different earth models: the FAI sphere or the WGS84 ellipsoid. **Flytec instruments** calculate distances using the **FAI sphere**, whereas some scoring programs like **Compe GPS Competition** use the **WGS84 ellipsoid**. Distances calculated between two points can disagree between these two models. Consequently, scoring programs that do not use the FAI sphere model may not exactly agree with the flight recorded by the instrument. This is of particular importance when determining if a pilot has crossed a start or turnpoint cylinder. For this reason the PWC recommends a tolerance of 0.5% on distance measurements to allow for the different earth models and calculation and rounding error.

Recommendation

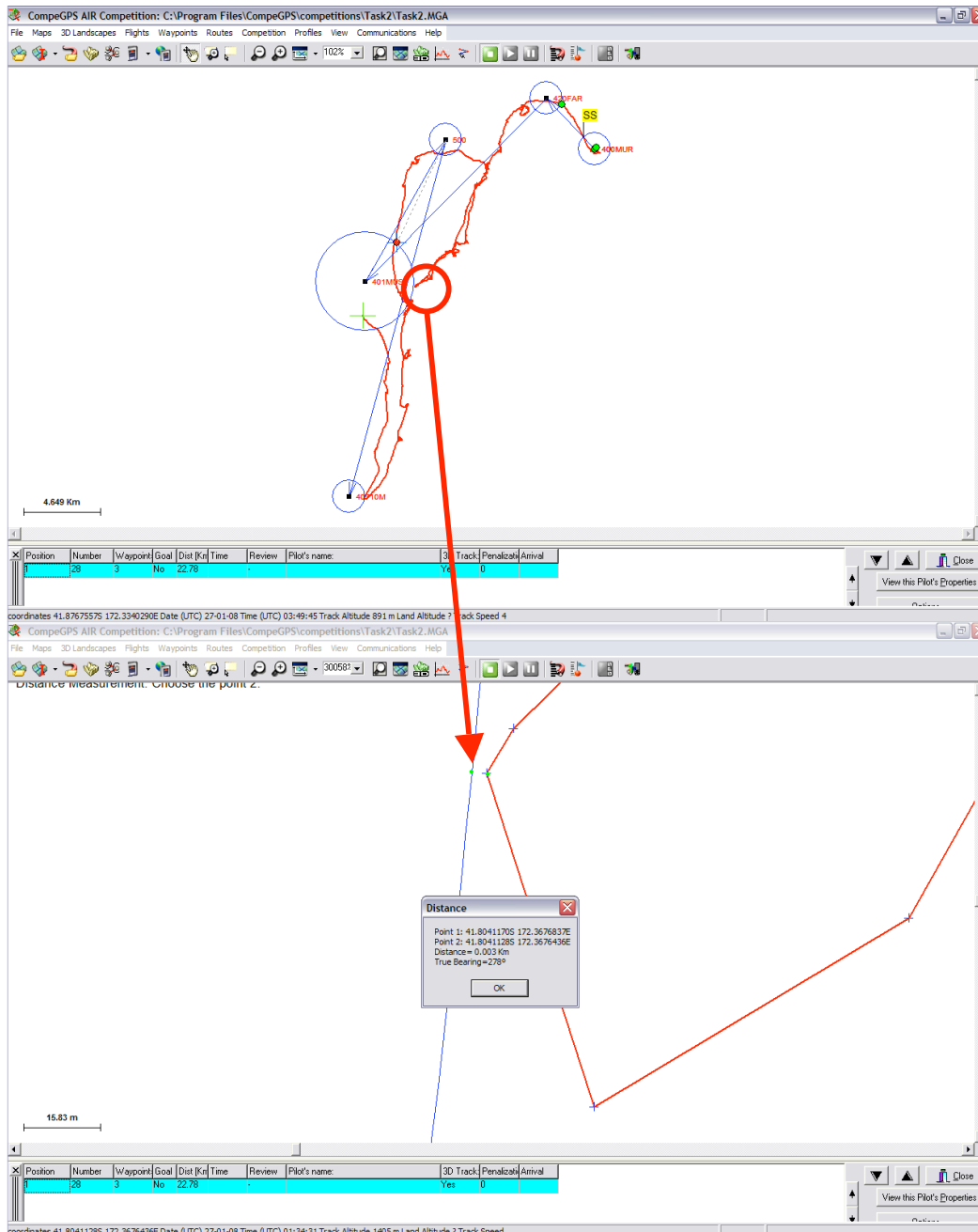
If this situation arises in a competition, where the pilot is certain that he had heard the "turnpoint reached" alert as he crossed the cylinder, and the scorekeeper does not grant this waypoint because the scoring program shows the pilot short of the cylinder by only a few meters, we recommend the pilot file a protest and that the distance is recalculated with this calculator <http://williams.best.vwh.net/gccalc.htm>.

Actual Example

During the New Zealand Nationals there was a scoring issue where the scoring program showed the pilot just short of making a turnpoint yet the pilot was sure that he heard the "turnpoint reached" alert. This prompted questions about how distances should be judged and what accuracy can be expected from the GPS and the scoring programs. The pilot's track log, as shown in CompeGPS, is below and it can be seen why the scorekeeper thought that the pilot did not make the turnpoint. However, when the FAI Sphere calculations are done (also shown below) you can see that the pilot was inside the cylinder and that the instrument correctly reported that the pilot had achieved the turnpoint.

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Flight as shown by CompeGPS:



As the pilot approached the cylinder, the pilot heard the typical “turnpoint reached” alert and, as is common practice, turned immediately. This is visible in the tracklog above by the two-tracklog points within one second of each other. These two points are automatically recorded by the 6030 when it recognizes that it is inside the cylinder. In this case all settings were correct, however, CompeGPS Air Competition shows the pilot missing the cylinder by 3 meters.

The discrepancy between the Flytec GPS and the CompeGPS Air results from a different earth model being used for the distance calculation as shown below.

Source <http://williams.best.vwh.net/qccalc.htm>

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Distances as shown by FAI Sphere and WGS84 Ellipsoid:

The screenshot shows the 'Javascript Great Circle Calculator' interface. The 'Input Data' section contains the following values:

Lat1	Lon1	Lat2	Lon2
41:48.089 S	172:19.903 E	41:48.247 S	172:22.061 E

The 'Output' section shows the following results:

Course 1-2	Course 2-1	Distance
95.6214705°	275.597496°	2.9956052181

The 'Distance Units' are set to 'km' and the 'Earth model' is set to 'FAI sphere'. A red circle highlights the 'FAI sphere' dropdown, and another red circle highlights the 'Distance' output value. A text box on the right states: 'According to the FAI Sphere the pilot was 4.5m inside the 3000m radius'.

The screenshot shows the same 'Javascript Great Circle Calculator' interface with the same input data as the first screenshot. The 'Output' section shows the following results:

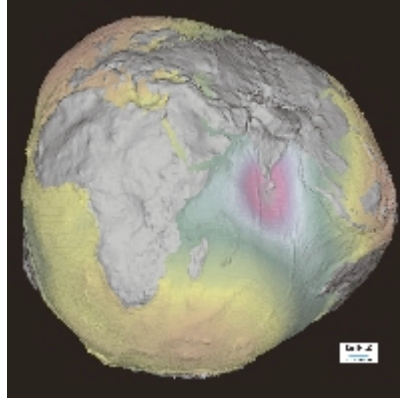
Course 1-2	Course 2-1	Distance
95.6006739°	275.576699°	3.003324141

The 'Distance Units' are set to 'km' and the 'Earth model' is set to 'WGS84/NAD83/GRS80'. A red circle highlights the 'WGS84/NAD83/GRS80' dropdown, and another red circle highlights the 'Distance' output value. A text box on the right states: 'The WGS84 Ellipsoid shows that he was 3m outside the radius'.

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The different earth models

The earth is not an exact sphere. Its' shape is more irregular and is referred to as a "Geoid". Source : <http://www.gfz-potsdam.de/news/foto/champ/welcome.html>



It is almost impossible to measure exact distances on such a surface. Therefore different models have been developed to calculate distances. The easiest way to calculate distances is to treat the earth as a sphere.

Distance based on the spherical model

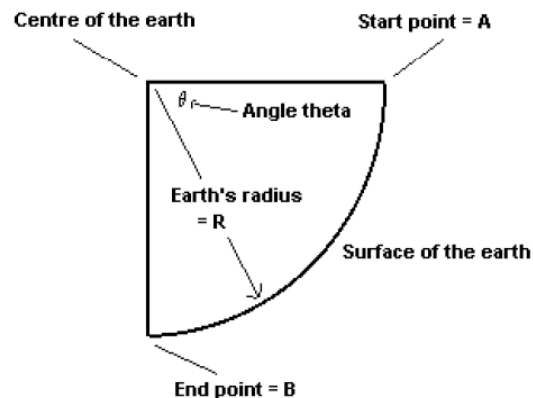
FAI proposed the so called "FAI sphere" with a radius of 6371km. In the FAI Sporting Code Section 7D Class O from 1st May 2004 FAI proposes a calculation algorithm based on the great circle distance.

CIVL - Section 7-D 1st May 2004

9 DISTANCE CALCULATION

The great circle distance is the shortest distance between two points on the earth's surface. The line between the two points is an arc on a circle in a plane defined by three points: the start point, the end point and the centre of the earth. The distance along this arc can be determined using spherical trigonometry using R, the earth's radius, and theta, the angle in the centre of the sector formed by the earth's centre, the start point and the end point.

Each degree of angle at the earth's centre, extended upward to the surface of the Earth, is equivalent to a distance of 1/360th of the circumference of the earth using the FAI radius given in paragraph 5.2.1.2. Since the circumference is equal to $2 \times \text{Pi} \times \text{Radius}$, $1 \text{ degree} = 2 \times \text{Pi} \times R / 360 = 111.194926642 \text{ km}$ at the earth's surface.



The angle between the two points is given by the following formula:
 $\text{Cos}(\text{theta}) = \text{Cos}(\text{LatA}) \times \text{Cos}(\text{LatB}) \times \text{Cos}(\text{LongA} - \text{LongB}) + \text{Sin}(\text{LatA}) \times \text{Sin}(\text{LatB})$

Radius = 6371 km. For calculation details see attachment.

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Distance based on the ellipsoid model

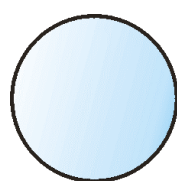
In the newer version of FAI sporting code from, 1st May 2005, chapter 9 states the following:

9 DISTANCE CALCULATION

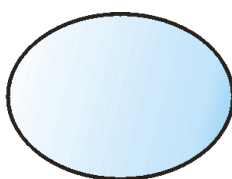
The preferred method of distance calculation is the GPS or approved flight data recorder, set to use the WGS84 datum. However, if you do not have such an instrument, and your flight is less than 100 km, you may use the following calculation method as an alternative.

Either scale the distance off the map directly using a ruler, or else locate the coordinates of the two points of interest. Find a navigational software program that will calculate the distance for you, for example the one available from the USDA at: <http://www.wcrl.ars.usda.gov/cec/java/lat-long.htm> [Unfortunately this link is broken. But the above mentioned calculator <http://williams.best.vwh.net/gccalc.htm> is also OK.]

CompeGPS Air Competition uses the WGS 84 Ellipsoid to calculate distances.



Sphere



Spheroid
(Ellipsoid)

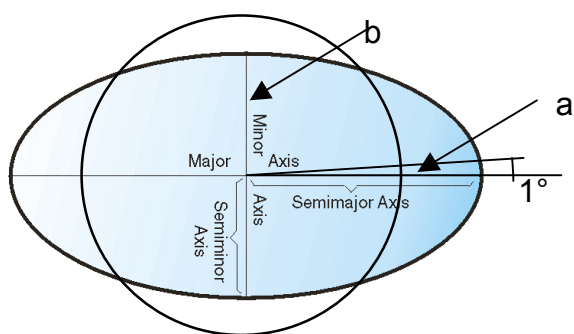
$$f = \frac{a - b}{a}$$

WGS84

a= 6378.137km

b= 6356.752315km

f=1/298.257223563



FAI Sphere

a=b=6371.000km

The major and minor axes of an ellipse

To calculate distances on such an Ellipsoid, Vicenty developed an algorithm in 1975 (See attachment). This algorithm is quite complex and a real burden for the small microprocessor found in a flight instrument. Please note that the CPU in a flight instrument, in addition to processing GPS functions, must also process flight data calculations (i.e., altitude, vario, TEC, STF, etc.). On the other hand, a PC running a program like CompeGPS can calculate ellipsoid distances relatively easy. There also several Java scripts available <http://williams.best.vwh.net/gccalc.htm> that can perform the calculation.

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Comparison of distance measurements

Differences are only a function of the Latitude. Because the Radius at the circumference (The equator) of the WGS 84 Ellipsoid is bigger than the FAI Sphere, the difference is the biggest if you fly 1° from south to north or vice versa. See picture above.

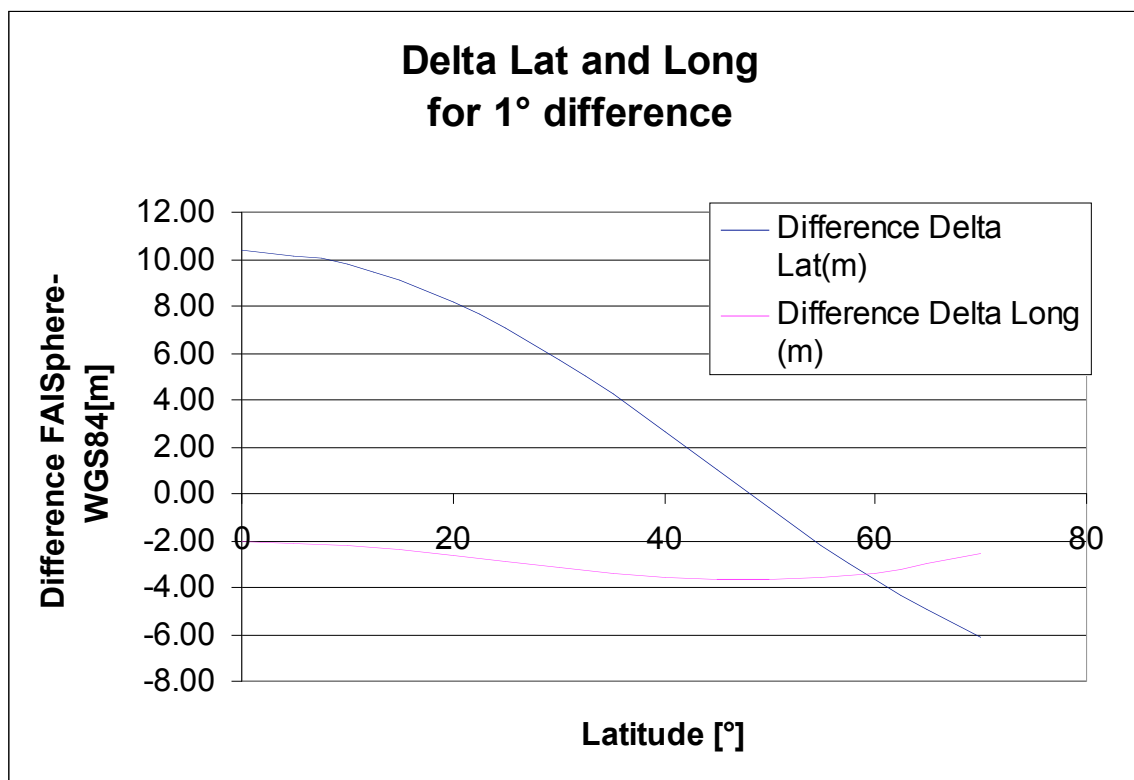
Distances for 1-minute difference in Latitude

Lat 1 [N]	Long 1 [E]	Lat 2 [N]	Long 2 [E]	FAI Sphere [km]	WGS 84 [km]	Difference Delta Lat(m)	Difference for a 400m FAI radius
0:00.000	0:00.000	0:01.000	0:00.000	1.85325	1.84290	10.34	2.23
10:00.000	0:00.000	10:01.000	0:00.000	1.85325	1.84346	9.79	2.11
20:00.000	0:00.000	20:01.000	0:00.000	1.85325	1.84507	8.18	1.76
30:00.000	0:00.000	30:01.000	0:00.000	1.85325	1.84754	5.71	1.23
40:00.000	0:00.000	40:01.000	0:00.000	1.85325	1.85058	2.67	0.58
50:00.000	0:00.000	50:01.000	0:00.000	1.85325	1.85382	-0.57	-0.12
60:00.000	0:00.000	60:01.000	0:00.000	1.85325	1.85687	-3.63	-0.78
70:00.000	0:00.000	70:01.000	0:00.000	1.85325	1.85937	-6.12	-1.32

Distances for 1-minute difference in Longitude

Lat 1 [N]	Long 1 [E]	Lat 2 [N]	Long 2 [E]	FAI Sphere [km]	WGS 84 [km]	Difference Delta Long (m)	Difference for a 400m FAI radius
0:00.000	0:00.000	0:00.000	0:01.000	1.85325	1.85532	-2.08	-0.45
10:00.000	0:00.000	10:00.000	0:01.000	1.82509	1.82732	-2.23	-0.49
20:00.000	0:00.000	20:00.000	0:01.000	1.74148	1.74412	-2.63	-0.60
30:00.000	0:00.000	30:00.000	0:01.000	1.60496	1.60810	-3.14	-0.78
40:00.000	0:00.000	40:00.000	0:01.000	1.41967	1.42323	-3.56	-1.00
50:00.000	0:00.000	50:00.000	0:01.000	1.19125	1.19493	-3.68	-1.24
60:00.000	0:00.000	60:00.000	0:01.000	0.92662	0.93000	-3.38	-1.46
70:00.000	0:00.000	70:00.000	0:01.000	0.63385	0.63644	-2.59	-1.64

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Conclusion

There are different methods to calculate distances on the earth's surface. In paragliding and hang gliding sports two systems are used, the (older) FAI sphere and the WGS84 ellipsoid. The WGS84 ellipsoid needs more calculation power than the FAI sphere. Therefore the FAI sphere method is better suited for flight instruments; whereas, PC based flight evaluation software use the WGS84 ellipsoid.

The FAI SPORTING CODE, (SECTION 7D, CLASS O, HANG GLIDERS and PARAGLIDERS RECORDS and BADGES CLASSES I / II / III / IV / V, Effective: 1st May 2005) states that the preferred method is the WGS84 system. However, it doesn't state that the FAI sphere isn't valid anymore.

The differences for a 400m FAI radius are below 3m and generally there should not be problem, however, care must be taken in the case of larger radii. In the situation where a competitor's track shows them just barely missing the cylinder, the scorekeeper should not only rely on the WGS84 ellipsoid provided by CompeGPS Air Competition, but also on a FAI sphere calculation from Internet resources like <http://williams.best.vwh.net/gccalc.htm> Furthermore, if a pilot believes that the scorekeeper has given a penalty for missing the cylinder by only a few meters, the pilot should file a protest and calculate the distance with the calculator above.

FAI/CIVL should clearly state what calculation method(s) will be accepted and should also state the tolerances that will be accepted or the procedure for scorekeepers in the event of a discrepancy between CompeGPS and the pilots flight instrument.

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Attachment

FAI Sphere

Source:

SPORTING CODE, SECTION 7D, CLASS O

HANG GLIDERS and PARAGLIDERS RECORDS and BADGES CLASSES I / II / III / IV / V

Effective: 1st May 2004

Chapter 9, page 26

Putting the above information together, using a more accurate formula for theta using sines instead of cosine, here

is a formula used for calculating the great circle distance when you know the latitude (Lat) and longitude (Long), in

decimal degrees, of the two points of interest A and B :

Distance = $2 * \text{Pi} * \text{Radius} / 360 * 2 * \text{Arcsin}(\text{Part4})$, where:

Part1 = $\text{Cos}(\text{LatA}) * \text{Cos}(\text{LatB})$

Part2 = $\text{Sin}((\text{LongA} - \text{LongB}) / 2)$

Part3 = $\text{Sin}((\text{LatA} - \text{LatB}) / 2)$

Part4 = $(\text{Part1} * \text{Part2}^2 + \text{Part3}^2)$

Radius = 6 371.0 km

Here is another formula, which is slightly more accurate (0.1km in a circumnavigation of the earth):

Distance = $2 * \text{Radius} * \text{Arcsin}(\text{Part4})$, where :

Part1 = $\text{Cos}(\text{LatB}) * \text{Sin}(\text{LongB}) - \text{Cos}(\text{LatA}) * \text{Sin}(\text{LongA})$

Part2 = $\text{Cos}(\text{LatB}) * \text{Cos}(\text{LongB}) - \text{Cos}(\text{LatA}) * \text{Cos}(\text{LongA})$

Part3 = $\text{Sin}(\text{LatB}) - \text{Sin}(\text{LatA})$

Part4 = $0.5 * \text{Square root}(\text{Part1}^2 + \text{Part2}^2 + \text{Part3}^2)$

Some calculators or computers may require that you enter the coordinates in radians rather than in degrees. In this case, use the following relationship : Radians = Degrees x Pi/180.

The earth is not exactly spherical, in fact it is oblate (fatter around the equator), but the FAI uses the above radius figure as an average radius for flights in all directions.

To check your formula, calculate the distance for a flight halfway around the world, ie. From 0°Lat, 0°Long to 0°Lat 180° Long. Y To check your use of the formula, calculate the distance for a flight halfway around the world, i.e. from 0°Lat, 0°Long to 0°Lat, 180°Long. You should obtain a distance of 20 015.09 km. For this you will need a calculator with a precision of at least 10 decimal places.

It is suggested that each NAC have a method for using the above formulae to verify pilots' claims.

Several computer programs are available for that purpose.

Also, see Appendix 2 of Annex C the IGC Section 3 of the Sporting Code

Vicenty's Algorithm

Source: <http://www.movable-type.co.uk/scripts/latlong-vincenty.html>

a, b = major & minor semiaxes of the ellipsoid

f = flattening $(a-b)/a$

ϕ_1, ϕ_2 = geodetic latitude

L = difference in longitude

$U_1 = \text{atan}((1-f) \cdot \tan \phi_1)$ (U is 'reduced latitude')

$U_2 = \text{atan}((1-f) \cdot \tan \phi_2)$

$\lambda = L, \lambda' = 2\pi$

while $\text{abs}(\lambda - \lambda') > 10^{-12}$ { (i.e. 0.06mm)

$$\sin \sigma = \sqrt{(\cos U_2 \cdot \sin \lambda)^2 + (\cos U_1 \cdot \sin U_2 - \sin U_1 \cdot \cos U_2 \cdot \cos \lambda)^2} \quad (14)$$

$$\cos \sigma = \sin U_1 \cdot \sin U_2 + \cos U_1 \cdot \cos U_2 \cdot \cos \lambda \quad (15)$$

$$\sigma = \text{atan2}(\sin \sigma, \cos \sigma) \quad (16)$$

$$\sin \alpha = \cos U_1 \cdot \cos U_2 \cdot \sin \lambda / \sin \sigma \quad (17)$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha \text{ (trig identity; §6)}$$

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$$\cos 2\sigma_m = \cos \sigma - 2 \cdot \sin U_1 \cdot \sin U_2 / \cos^2 \alpha \quad (18)$$

$$C = f/16 \cdot \cos^2 \alpha \cdot [4 + f \cdot (4 - 3 \cdot \cos^2 \alpha)] \quad (10)$$

$$\lambda' = \lambda$$

$$\lambda = L + (1 - C) \cdot f \cdot \sin \alpha \cdot \{ \sigma + C \cdot \sin \sigma \cdot [\cos 2\sigma_m + C \cdot \cos \sigma \cdot (-1 + 2 \cdot \cos^2 2\sigma_m)] \} \quad (11)$$

}

$$u^2 = \cos^2 \alpha \cdot (a^2 - b^2) / b^2$$

$$A = 1 + u^2 / 16384 \cdot \{ 4096 + u^2 \cdot [-768 + u^2 \cdot (320 - 175 \cdot u^2)] \} \quad (3)$$

$$B = u^2 / 1024 \cdot \{ 256 + u^2 \cdot [-128 + u^2 \cdot (74 - 47 \cdot u^2)] \} \quad (4)$$

$$\Delta \sigma = B \cdot \sin \sigma \cdot \{ \cos 2\sigma_m + B/4 \cdot [\cos \sigma \cdot (-1 + 2 \cdot \cos^2 2\sigma_m) - B/6 \cdot \cos 2\sigma_m \cdot (-3 + 4 \cdot \sin^2 \sigma) \cdot (-3 + 4 \cdot \cos^2 2\sigma_m)] \} \quad (6)$$

$$s = b \cdot A \cdot (\sigma - \Delta \sigma) \quad (19)$$

$$\alpha_1 = \text{atan2}(\cos U_2 \cdot \sin \lambda, \cos U_1 \cdot \sin U_2 - \sin U_1 \cdot \cos U_2 \cdot \cos \lambda) \quad (20)$$

$$\alpha_2 = \text{atan2}(\cos U_1 \cdot \sin \lambda, -\sin U_1 \cdot \cos U_2 + \cos U_1 \cdot \sin U_2 \cdot \cos \lambda) \quad (21)$$

Where:

- s is the distance (in the same units as a & b)
- α_1 is the initial bearing, or forward azimuth
- α_2 is the final bearing (in direction $p_1 \square p_2$)

Note: Vincenty observes that eqn. (18) becomes indeterminate over equatorial lines (since $\cos^2 \alpha \square 0$); in this case, set $\cos 2\sigma_m$ to 0 and the result is computed correctly. He also points out that the formula may have no solution between two nearly antipodal points; an iteration limit traps this case.